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Experiments on transient natural convection in a cavity

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A laboratory experiment is used to study the transient flow in an initially isothermal cavity at temperature T_0 following the rapid change of the two vertical endwalls to temperatures $T_0 \pm \Delta T$ respectively. Individual temperature records are taken and the transient flow in the entire cavity is visualized with the aid of a tracer technique. It is shown that an oscillatory approach to final steady-state conditions exists for certain flow regimes, although the form of the oscillatory response is different to that suggested by previous work. It is argued that this oscillatory behaviour is due to the inertia of the flow entering the interior of the cavity from the sidewall boundary layers, which may lead to a form of internal hydraulic jump if the Rayleigh number is sufficiently large.

1. Introduction

Convection in cavities is an area of study with applications in a number of diverse fields: the growth of crystals for the semiconductor industry, thermal insulation in buildings; convective processes in lakes, estuaries and the oceans; and convection in magma chambers; to name but a few. The majority of prior work on cavity convection has been concerned with steady-state situations. Yet in many of the fields of application listed above, the convective flows may be in a transient or unsteady state. Recognizing this fact, some of the recent work in the field has focused on the nature of the flow in the transient regime and the manner in which this flow evolves into the final steady state.

The most fundamental question in regard to the transient flow is how long does it take for the flow to reach steady state? Patterson & Imberger (1980) (hereinafter referred to as PI) considered an initially isothermal cavity, at temperature T_0, with insulated top and bottom boundaries, and conducting vertical endwalls brought to temperatures $T_0 \pm \Delta T$ respectively at time $t = 0$. Utilizing scaling arguments, they predicted that the cavity flow reached steady state in a timescale $\tau_f \sim hl/\kappa Ra^{\frac{1}{4}}$. In this expression h and l are respectively the depth and length of the cavity, and the Rayleigh number $Ra = g\alpha\Delta T h^3/\nu\kappa$. If the boundary conditions are changed on a timescale less than τ_f then clearly the flow never reaches steady state. The timescale τ_f was consistent with their own numerical experiments, and has since been confirmed by laboratory experiments over different ranges of the relevant dimensionless parameters (Ra and $A = h/l$) by Yewell, Poulikakas & Bejan (1982) and Hamblin & Ivey (1983).

A second, and potentially very important, feature of some of the numerical experiments of both PI and Gresho *et al.* (1980) was the existence of an oscillatory approach to the final steady state. In the application to the crystal-growth process, for example, this importance stems from the fact that these systems are often in a transient state (Pimputkar & Ostrach 1981) and thermal oscillations can induce undesirable striations in the growing crystals. In environmental flows, the oscillatory behaviour may well be important in determining the dilution levels of pollutants discharged into natural water bodies, for example. While a feature of the numerical experiments, neither of the laboratory experiments discussed above exhibited any form of oscillatory approach to the final steady state.

Patterson (1983) has recently proposed an extensive classification scheme of these transient flows and has shown that these laboratory experiments were performed in a flow regime where oscillatory behaviour was actually not to be expected. The purpose of the present work is to study in a simple laboratory experiment transient convection flows in a flow regime where an oscillatory approach to steady state may be expected. In particular, the aim of the experiments is to examine the nature of the evolving temperature and flow fields in such a regime for times small compared to the steady-state time.

2. Experiments

When do we expect to see an oscillatory approach to steady state? After bringing the endwalls to temperatures $T_0 \pm \Delta T$, thermal boundary layers (of thickness $\delta \sim h/Ra^{\frac{1}{4}}$ and with characteristic vertical velocities $w \sim \kappa Ra^{\frac{1}{2}}/h$ when $Pr > 1$) form on the vertical endwalls in a timescale given by (PI)

$$\tau_b \sim h^2/\kappa Ra^{\frac{1}{2}}. \tag{1}$$

These vertically moving buoyant flows are turned by the presence of the top and bottom boundaries and flow horizontally out into the cavity interior. PI and Patterson (1983) argue that these intrusions cross the cavity and may 'pile up' at the far boundary. A resulting tilting of the isotherms beyond the horizontal occurs, and a cavity-scale oscillation or seiching of the isotherms may ensue. If the Prandtl number $Pr > A^{-6}$ (as is the case in the experiments discussed in this paper) the oscillatory behaviour is predicted in the flow regime where

$$Ra > Pr^4 A^{-4}. \tag{2}$$

In order to be in a flow regime where an oscillatory response is to be expected, we must not only satisfy (2) but also attempt to satisfy the initial conditions under which these responses have been observed in the numerical experiments. In particular, the endwalls are brought to their final steady temperatures instantaneously in the numerical experiments. In the laboratory it takes a finite time τ_w for the wall to reach the final temperature. It seems reasonable to demand, then, that such an externally imposed forcing timescale be shorter than any natural timescales of the convective flows. The shortest natural timescale is the boundary-layer development timescale τ_b, given by (1), and one of the design criteria for the experiments is that $\tau_w < \tau_b$. The timescale τ_b may be interpreted as the time it takes for buoyant fluid to move through the boundary layer (i.e. $\tau \sim h/w$). The condition $\tau_w < \tau_b$ may therefore be physically interpreted as meaning that the flow on the two vertical boundary layers is fully developed before there is any ejection of fluid into the interior of the cavity. Earlier laboratory experiments measured heat-transfer rates but did not satisfy $\tau_w < \tau_b$. Both the transient nature of the flow and the need to satisfy this timescale condition preclude the direct measurement of any heat-transfer rates in the present experiments. This is a regrettable shortcoming of the experiments, but is accepted because for the first time the experiment allowed an examination in the laboratory of the transient flow in a regime where an oscillatory response is to be expected.

The experiments were performed in a tank with glass walls and bottom in order to visualize the flow. A styrofoam lid was added to yield a working space of $h = l = 24$ cm. The width of the tank was 20 cm. The two endwalls consisted of copper plate, each approximately 1 mm thick. These plates separated the working space from the two end water jackets. An experiment was prepared by filling the working space with filtered, deaerated fluid, emptying the two water jackets, and allowing the whole system to come to an isothermal condition overnight at temperature T_0. All experiments were done in a temperature controlled laboratory with air temperatures maintained at T_0.

In two external temperature-controlled baths, water was brought to the desired end temperatures $T_0 \pm \Delta T$. The temperature difference ΔT was nominally 5.0 °C. The experiment was started by simultaneously pouring the water from these two baths into the respective end water jackets. The temperatures in the jackets were subsequently maintained to within about 0.1 °C of their respective values. This was accomplished by placing an immersion heater into the hot-water jacket and by stirring vigorously while pumping cold water from the external cold bath through a cooling coil placed in the cold-water jacket. This combination of pouring water (about 5 s to fill the water jackets) already at the desired temperature, and the rapid heat transfer through the thin copper endwalls (the conduction timescale is of order 10 ms) enabled the endwalls to be brought to temperature in timescales $\tau_w < \tau_b$ (see below) as desired. Two different working fluids were used: a mixture of glycerol and water with a Pr of order 80 (Segur 1953), and pure water. The use of these two fluids made it possible to vary both the Prandtl number and the Rayleigh number in the experiments.

Temperatures, obtained with a pair of thermistors mounted on the end of fine stainless steel rods inserted through the lid, were recorded continuously on a chart recorder. Thermistor positions were chosen so that they would be away from both the direct influence of the sidewall boundary layers and the central nodal position that might be expected if there were basin-scale standing waves dominated by the first mode. The scaling arguments of PI predicted the thickness of the first outflows into the cavity, and the vertical positions of the thermistors were then chosen so that they would sample within these outflows.

Flow visualization was accomplished by seeding the flow with a commercially available tracer. The tracer consisted of a concentrated suspension of tiny (approximately $6 \times 30 \times 0.07$ μm), transparent, crystalline platelets with an essentially negligible settling velocity, even in quiescent water. Illuminating the particles through a narrow slit along the longitudinal centreline of the glass bottom of the tank enabled the visualization of the flow in a sheet of fluid about 1 cm wide and 24 cm long and high (i.e. the full cavity). Long photographic exposures (typically 15 s) produced streak photographs of the evolving flow field throughout the entire tank.

3. Results

Let us first consider the temperature records. Figure 1(*a*) shows the temperature trace for an experiment with the glycerol–water mix for which the transition parameter, from (2), has the value $Ra\, Pr^{-4} A^4 = 8.6$. That is, the parameter just exceeds the critical value of unity. For this experiment τ_b is equal to 27 s, from (1),

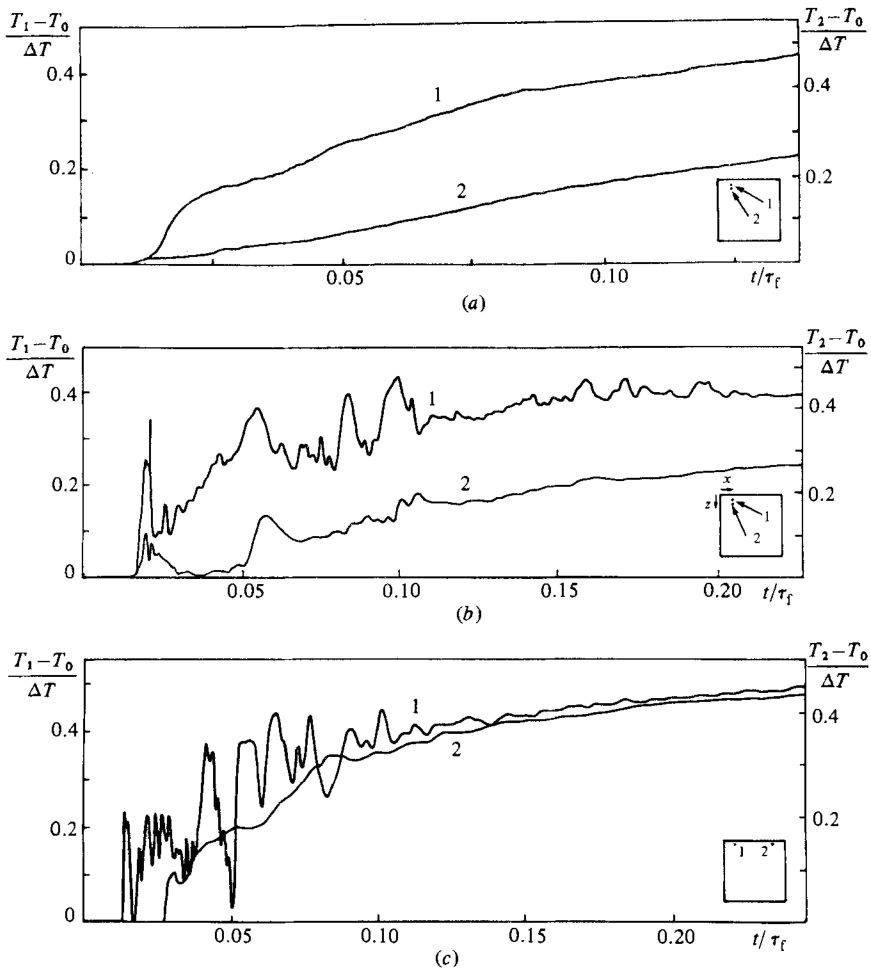


FIGURE 1. (a) Temperature traces for an experiment with $A = 1.0$, $Ra = 3.9 \times 10^8$, $Pr = 82$; $\tau_f = 3.83 \times 10^3$ s, $\Delta T = 5.0$ °C. The insert sketch shows the location in the cavity of the thermistors producing each trace. (Coordinate orientation is shown in (b).) Trace 1: $x/l = 0.21$, $z/h = 0.033$. Trace 2: $x/l = 0.21$, $z/h = 0.063$. (b) Temperature traces for an experiment with $A = 1.0$, $Ra = 9.2 \times 10^8$, $Pr = 7.1$; $\tau_f = 2.33 \times 10^3$ s, $\Delta T = 4.8$ °C. Trace 1: $x/l = 0.21$, $z/h = 0.033$. Trace 2: $x/l = 0.21$, $z/l = 0.063$. (c) Temperature traces from an experiment with $A = 1.0$, $Ra = 1.2 \times 10^9$, $Pr = 6.6$; $\tau_f = 2.15 \times 10^3$ s, $\Delta T = 5.0$ °C. Trace 1: $x/l = 0.21$, $z/h = 0.033$. Trace 2: $x/l = 0.79$, $z/h = 0.033$.

and is thus considerably longer than $\tau_w = O(5\text{ s})$. Figure 1(a) clearly illustrates a sudden jump in temperature (as the buoyant outflow first reaches the thermistor location), followed by a fairly smooth asymptotic rise of the two temperature traces towards their steady-state values. There is no evidence of any marked oscillatory behaviour.

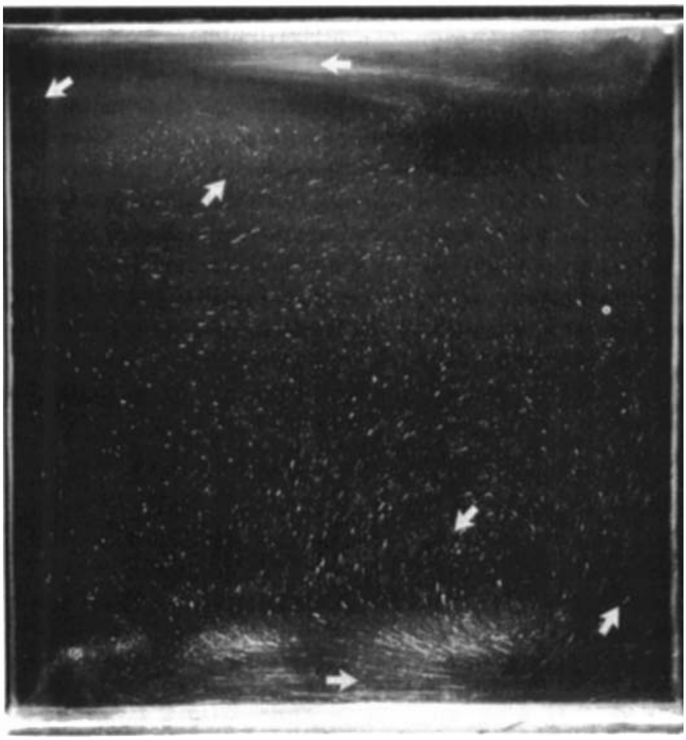
Figure 1(b), on the other hand, from an experiment with water and with $Ra\,Pr^{-4}A^4 = 3.6 \times 10^5$, illustrates a highly oscillatory temperature signal superimposed upon the asymptotic approach to the final steady state. The maximum amplitude of the fluctuations are of order 1 °C, a substantial fraction of the endwall temperature difference $\Delta T = 4.8$ °C. There is some suggestion of a weak correlation between the two records for the largest-scale fluctuations, but in general the records appear very noisy and uncorrelated. Oscillatory behaviour disappears for times greater than about $0.2\tau_f$. Finally, figure 1(*c*) illustrates the striking attenuation of the temperature fluctuations with increasing distance from the source of the outflow in the endwall boundary layers. These measurements suggest that any oscillatory motion producing these temperature fluctuations cannot be regular basin-scale standing internal waves. Some insight into their character can be gained from the flow-visualization experiments.

Figure 2 contains a series of streak photographs showing the transient flow field for an experiment in the parameter range in which oscillatory temperature responses were observed. In the experiment shown, the parameter $Ra\,Pr^{-4}A^4$ in (2) has a value of 3.6×10^5, and the hot wall is on the right-hand side in all figures. While there is a general symmetry in each picture, the most striking features of the flow field are both its complexity and the rapidity with which it evolves. Strong horizontal hot and cold intrusions traverse the cavity adjacent to the top and bottom boundaries respectively. From PI, the timescale for the first intrusion to cross the cavity should be $T_g \sim lh/\kappa\, Pr^{\frac{1}{3}}\, Ra^{\frac{5}{12}} \sim 40$ s. Since $t = 60$ s in figure 2(*a*), the first intrusion should have traversed the cavity – as indeed the figure suggests. In response to the movement of these intrusions across the top and bottom of the cavity, the interior of the cavity responds rapidly.

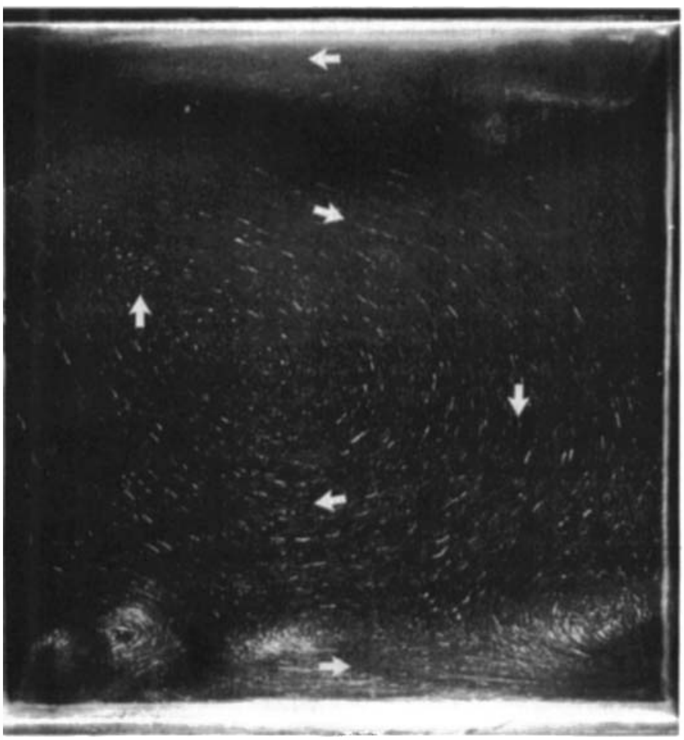
Initially in figure 2(*a*) one can identify two anticlockwise eddies in the upper left- and lower right-hand regions. In figure 2(*c*), some 40 s later, the interior is dominated by a single cavity-scale clockwise eddy. Twenty seconds later (figure 2*d*), this circulation is already breaking up, and in figure 2(*e*) a large anticlockwise eddy appears in the centre of the cavity. It should be emphasized that the flow evolves more rapidly than the time it takes for a fluid particle to complete one circuit of such large cavity-scale eddies as observed in figure 2(*c*), for example. The flow subsequently evolves somewhat more regularly, until one sees the near-steady picture in figure 2(*f*), with a uniform flow from right to left in the upper half of the cavity and a similar reverse flow in the lower half of the cavity.

Of particular interest, however, are the flow features in the upper right-hand and lower left-hand corners, where the respective hot and cold intrusions emerge from the vertical boundary layers. In figure 2(*c*) and in the close-up shown in figure 2(*g*), for example, we see a wave-like structure and, associated with it, vigorous small-scale closed eddies. There is no evidence of these structures for long time (figure 2*f*), and while they clearly evolve in time, they are present throughout the early figures. Most importantly, they are always found in the region of the emerging flow in the corner regions – regions with high temperature gradients.

These flow features are not found in the experiments done with the glycerol–water mix. Figure 3, for example, contains streak photographs for one of these experiments. Figure 3(*b*), taken at $t = 540$ s, does show one new feature not present in figure 2(*f*): a distinct 'lobe-like' recirculation area adjacent to the endwall regions. This may well be related to the fact that the viscous boundary layer surrounding the thermal boundary layer is now much thicker than for figure 2(*f*) (from PI the viscous boundary-layer thickness $\delta_\nu \sim Pr^{\frac{1}{2}}\, h/Ra^{\frac{1}{4}} = O(1.5\text{ cm})$ for this experiment). It is important to note there is no evidence of a stationary wave-like structure with associated eddies in the emerging intrusions in these experiments. It seems plausible, then, that the observed fluctuating temperature behaviour is related to the occurrence and persistence of these wave-like features and their associated small-scale eddies.

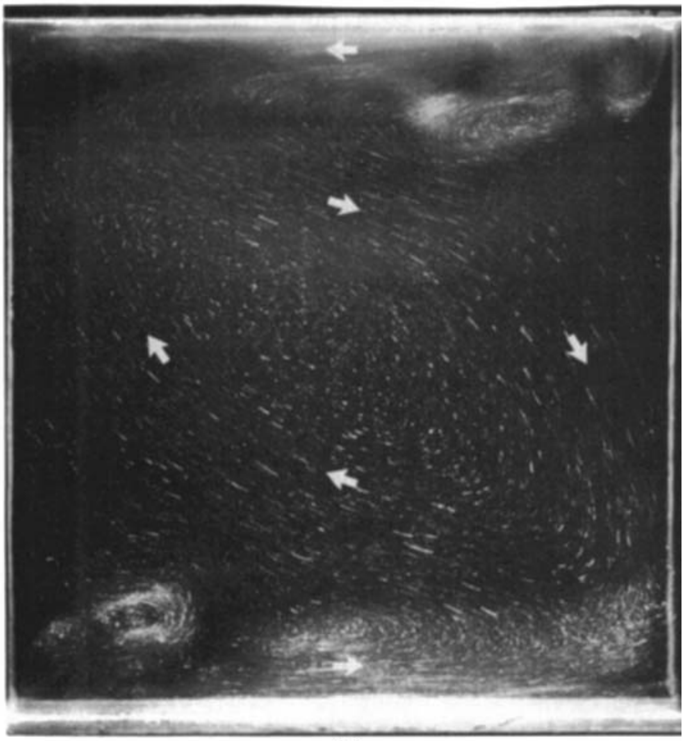


(a)



(b)

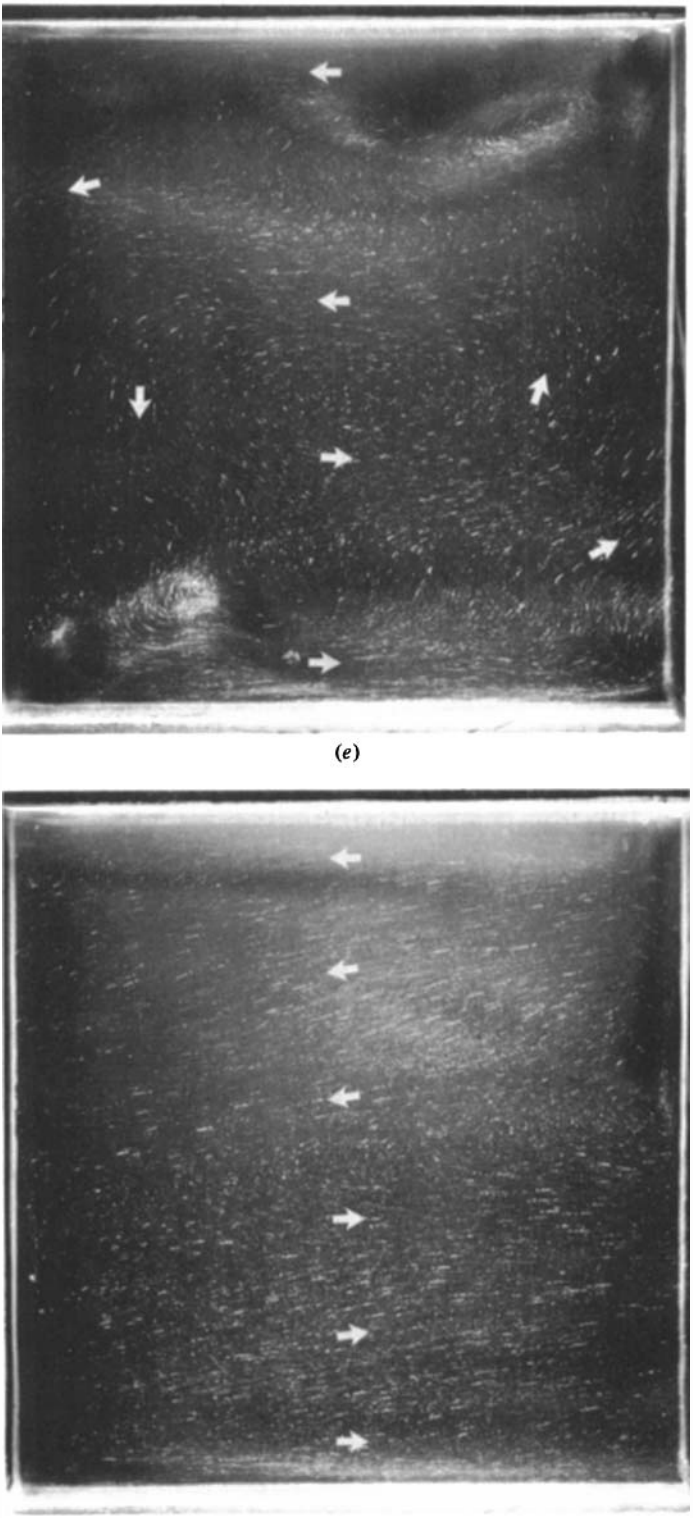
FIGURE 2 (a, b). For caption see p. 397.



(c)

(d)

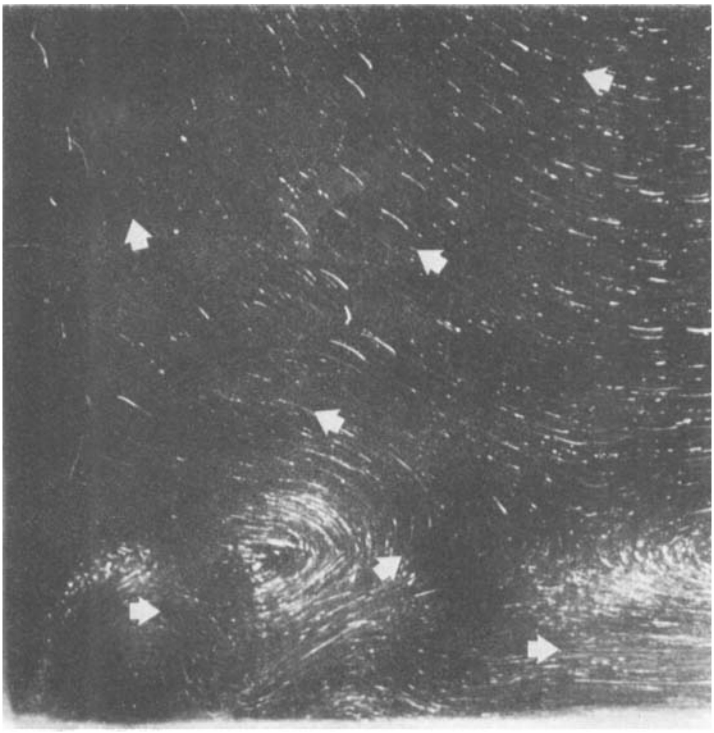
FIGURE 2(c, d). For caption see p. 397.



(e)

(f)

FIGURE 2(e, f). For caption see facing page.



(g)

FIGURE 2. Streak photographs of the transient flow for an experiment with $A = 1.0$, $Ra = 9.2 \times 10^8$, $Pr = 7.1$. The hot wall is on the right. The time to steady state $\tau_f \sim hl/\kappa Ra^{\frac{1}{4}} \approx 2.3 \times 10^3$ s. Each photograph is a 15 s exposure commencing at the time shown below. The only exception is (*f*) which is a 30 s exposure. (*g*) is a close-up of the lower left-hand corner of (*b*) showing the detail of the emerging flow. The horizontal scale of the photograph is about 7.5 cm. The arrows indicate local flow direction. (*a*) time = 60 s; (*b*) 80 s; (*c*) 100 s; (*d*) 120 s; (*e*) 140 s; (*f*) 510 s; (*g*) 80 s.

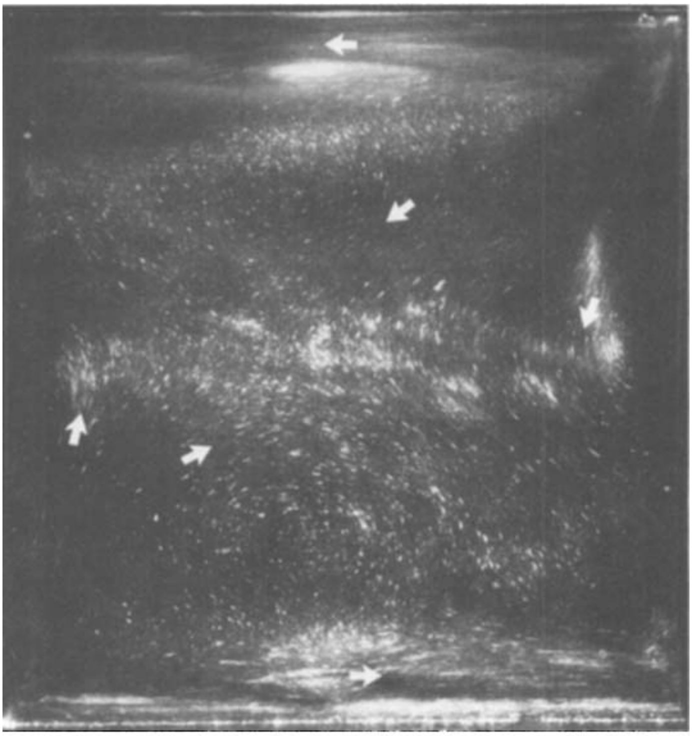
4. Discussion of results

While no single analysis could hope to provide a complete description of the complex transient flow revealed by these observations, simple scaling arguments do provide some insight into this flow. Let us examine, for example, the buoyant flow rising up the hot wall. (Owing to the symmetry of the cavity, we need to examine the flow along only one vertical wall.) As prior work has shown, at steady state and for $Pr > 1$ this boundary-layer flow has a characteristic horizontal thickness $\delta \sim h/Ra^{\frac{1}{4}}$, and within that boundary layer a characteristic vertical velocity $w \sim \kappa Ra^{\frac{1}{2}}/h$. Consider now the corner region where this rising flow is first turned owing to the presence of the insulated lid. Let us denote the horizontal velocity of the intrusion at this point by u, and the local vertical thickness of the intrusion by Δ. At the point where the rising flow is *first* turned, then, assuming constant pressure on the dividing streamline of the emerging flow, conservation of mass implies that

$$u \sim \frac{\kappa Ra^{\frac{1}{2}}}{h}, \quad \Delta \sim \frac{h}{Ra^{\frac{1}{4}}}. \tag{3}$$

We may define a characteristic internal Froude number for this horizontal flow by

$$F = u/(g'\Delta)^{\frac{1}{2}}, \tag{4}$$



(a)

(b)

FIGURE 3. Streak photographs of the transient flow for an experiment with $A = 1.0$, $Ra = 3.8\times10^8$, $Pr = 78$. The hot wall is on the right and the time to steady state $\tau_f \sim hl/\kappa Ra^{\frac{1}{4}} \approx 3.9\times10^3$ s. (a) is a 15 s exposure and (b) is a 30 s exposure. The arrows indicate local flow direction. (a) time = 80 s; (b) 540 s.

where u is the characteristic velocity scale over the thickness Δ. Substituting (3) into (4), and noting that $g' = (Ra\, \nu\kappa/h^3)$, we obtain

$$F \sim Ra^{\frac{1}{8}}/Pr^{\frac{1}{2}}. \tag{5}$$

Equation (5) thus provides an order-of-magnitude estimate of the internal Froude number F. The flow is critical when $F = 1$ (Turner 1973, p. 65). For $F > 1$ the flow is supercritical and, under these circumstances, may undergo an internal hydraulic jump with an increase in depth of the flowing layer as it moves from supercritical to subcritical flow. Indeed the flow features in the corners in figure 2 appear remarkably like internal hydraulic jumps. In the experiments discussed here, for the stable experiment shown in figure 1 (*a*) we obtain from (5) that $F = O(1.7)$, and for the unstable experiments shown in figures 1 (*b*) and 2 that $F = O(5)$. These are only order-of-magnitude estimates of F, but they do seem consistent with the idea that the instabilities are due to the occurrence of internal hydraulic jumps. These jumps will occur when the flow is supercritical, and from (5), that occurs when $Ra > Pr^4$.

In regard to the exact nature of this flow transition, we would expect, by analogy with open-channel flow (Lighthill 1978) that the energy loss associated with any internal hydraulic jump would be dissipated in a stationary wavetrain downstream of the jump for $1.0 < F < 1.3$. For higher values of the Froude numbers, these waves break, resulting in a disordered interface downstream. For the experiment in figure 2, we obtain from (5) that $F = O(5)$. Such a high value of the Froude number would imply there is indeed some entrainment at the jump (Turner 1973, p. 182; Wilkinson & Wood 1971). While it is difficult to see any direct evidence of this entrainment in the close-up of the jump in figure 2(*g*), owing in part to the relatively small horizontal scale of the jump, the vigorous small-scale eddies in the region can be clearly identified. Some entrainment in this region is consistent, however, with the observation of rapid, disordered temperature fluctuations within the outflow seen in figure 1 (*b*).

These observations suggest the following scenario. The first intrusions to traverse the cavity are supercritical as they emerge from the vertical boundary layer. The internal Froude numbers of these emerging flows may be large and the flow undergoes an internal jump with some associated entrainment at the jump location. The presence of the jump and the associated mixing activity is responsible for the high-frequency disordered temperature fluctuations observed in the experiment. This situation is unsteady, however, owing to the finite size of the cavity. As time proceeds and further buoyant fluid is ejected from the vertical boundary layers, the interior is set into motion and stratifies in temperature. In terms of the horizontal intrusive flow, the flowing depth increases with time and the jump is effectively flooded.

It is possible to make a rough estimate of when that time occurs by considering the following simplified picture of the core motion. When $Ra > A^{-4}$ (true for all the present experiments), it has been observed in previous work (Bejan, Al-Homoud & Imberger 1981; Yewell *et al.* 1982; Hamblin & Ivey 1983) that, when the flow reaches steady state, at time τ_f, the interior will have a near-linear vertical temperature stratification. The inequality $Ra > A^{-4}$, first derived by Bejan *et al.* (1981), may be interpreted as follows: if $Ra > A^{-4}$, vertical diffusion of heat, on timescale $T_d \sim h^2/\kappa$, is too slow to smear the vertical temperature gradients before the flow reaches steady state. We can thus think of the interior stratification as being established by 'thermal fronts' which advance into the cavity from the top and bottom boundary, with a characteristic vertical velocity $w_f \sim h/\tau_f \sim \kappa\, Ra^{\frac{1}{4}}/l$.

Consider now the flow at the jump discharging into the interior region. The exact form of downstream control for these jumps is not clear. For the purpose of illustration, however, we know for the case of a flowing upper layer and a stagnant lower layer confined between impermeable boundaries a constant distance apart, the flow will jump to a new flowing depth downstream Δ_2 given by (Turner 1973)

$$\frac{\Delta_2}{\Delta} = \tfrac{1}{2}[-1+(1+8F^2)^{\frac{1}{2}}]. \tag{6}$$

Using (5), in the limit of large F (6) yields

$$\frac{\Delta_2}{\Delta} \approx \sqrt{2}\frac{Ra^{\frac{1}{8}}}{Pr^{\frac{1}{2}}}.$$

This jump will flood when $\Delta_f = w_f t > \Delta_2$, and, non-dimensionalizing by τ_f, this yields a timescale of

$$\frac{t}{\tau_f} > \frac{\sqrt{2}}{Pr^{\frac{1}{2}}Ra^{\frac{1}{8}}}. \tag{7}$$

For the experiment in figure 1 (*b*), for example, $Pr = 7.1$ and $Ra = 9.2\times 10^8$, which yields $t/\tau_f > 0.04$ from (7). We see from figure 1 (*b*) that the oscillations persist for times up to $t/\tau_f \approx 0.2$. These arguments assume that (6) may be applied and, while they grossly simplify the complex core motion, they are in reasonable agreement with the observations of the flow.

Entrainment in the vicinity of the jump, where the flow emerges from the vertical boundary layers, could well lead to local fluctuations in the convective heat transfer. However, such fluctuations in heat-transfer rates must be localized, and these experiments do not explain the regular oscillations of heat-transfer rate at the midpoint of the cavity reported by PI. For their first series of numerical experiments with $Pr = 7$, the criterion $Ra > Pr^4$ does predict the demarkation between the steady and oscillatory solutions – although not the form of the response. The difficulty with further laboratory work lies in a means of measuring heat-transfer rates. Clearly, there are uncertainties as to how a numerical calculation would handle both high-frequency temperature oscillations and the supercritical flow suggested here. It would seem, however, that further insight into these phenomena might best be gained by numerical studies using some of the insights gained from the present study.

5. Conclusions

Transient natural convection in a cavity was studied in a simple laboratory experiment by rapidly bringing the conducting endwalls to their steady-state values. While published numerical solutions of these flows had suggested the existence of regular cavity-scale internal waves, no evidence for the existence of these waves could be found in the present laboratory experiments. An oscillatory approach to the final steady-state temperature distribution was observed, however, in the corner regions where the sidewall boundary layers discharged into the cavity. Temperature oscillations were of relatively large amplitude ($T'/\Delta T \approx 0.2$); were of a high frequency and disordered; and were attenuated rapidly with increasing distance from the source of outflow and decayed in times $t/\tau_f \approx 0.2$.

All of these observations can be explained by considering the inertia of the flow in the corner regions. It is argued that, following the rapid initialization of the endwall temperatures in timescales $\tau_w < \tau_b$, when $Ra > Pr^4$ the flow leaving the sidewall boundary layers is supercritical and an internal hydraulic jump with some associated mixing may occur. Although the design of the experiment precluded direct measurements of the heat-transfer rate, the fact that these jumps were observed only in the corner regions implies that any fluctuations in the heat-transfer rate would be most pronounced in those regions and would attenuate rapidly with increasing distance from the sidewalls. It would seem that further insight into these effects would best be gained by numerical studies.

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